Article

Transactions of the Institute of Measurement and Control
1–9

DOI: 10.1177/0142331215619973
tim.sagepub.com

Smith predictor based sliding mode control for a class of unstable processes

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Abstract

A strategy to regulate unstable processes using a modified Smith predictor based sliding mode controller (SP-SMC) is illustrated. The proposed scheme presents disturbance rejection and optimal control input usage with overall improved regulatory performances. The unstable process with time delay is first estimated using a simple measurement of limit cycle output obtained from a modified relay experiment. Then this paper extends a work on SP-SMC for unstable processes, which leads to significant improvements in its regulatory capacities of reference inputs and disturbances. A new control law is incorporated in the discontinuous part of sliding mode control such that the overall performance improves significantly. The metaheuristic search algorithm with some modifications has been implemented successfully to satisfy the new control performance index. The robustness of the controller is also tested under the process uncertainty. Illustrative examples show the simplicity and superiority of the presented design method over previously published approaches.

Keywords

Unstable process with time delay, sliding mode control, Smith predictor, robustness, control input

Introduction

Compared with stable processes, unstable processes are more difficult to control. Time-delay is another factor which is commonly encountered in biological, chemical, electronic and mechanical systems. There are some unstable processes in industry, such as chemical reactors, and their stabilization is essential for successful operations. Especially, unstable processes coupled with time-delay makes control system design a difficult task, which has attracted attention from the control community (Sree and Chidambaram, 2005). Techniques have been reported to improve control of unstable delay processes, such as proportional-integral-derivative (PID) tuning (Ajmeri and Ali, 2015; Hwang and Hwang, 2004; Lee and Wang, 2010; Shamsuzzoha and Skogestad, 2010), PI-PD tuning (Majhi and Atherton, 2000b; Nema and Padhy, 2015) and Smith predictors (Majhi and Atherton, 2000a; Matausek and Micic, 1996). In reality, sometimes PID tuning rules are insufficient to overcome disturbance problems and process perturbation. Another simple and powerful control technique is the Smith predictor (SP). The controller can be designed as if the system is delay free. The improved structure of the SP to control open loop unstable processes has recently been shown in the literature (Matausek and Ribic, 2012; Padhan and Majhi, 2012). They have incorporated an additional filter in series with the PID controller to improve the regulatory performance.

The robustness to uncertainties becomes a main point in designing any control systems. As such, active research is continuing to develop controllers which can work successfully in spite of uncertainties. Sliding mode control (SMC) has evolved to deal with uncertainties. The SMC was not known to the control community at large until an article discussed by Utkin (1997). Basically a known SMC is a particular type of Variable Structure Control System (VSCS). It is a powerful robust nonlinear control technique that has been intensively accredited as one of the key approaches for the systematic design of robust controllers for systems operating under conditions of uncertainty (Bandyopadhyay et al., 2009). Literature has been presented for SMC robustness against disturbances and parameter variations. This type of technique is nowadays widely used in a variety of application areas like robotics, process control, aerospace and power electronics (Bandyopadhyay et al., 2009; Perruquetti and Barbot, 2002; Utkin, 1997).

A SMC in process control has been discussed more in Camacho and Rojas (2000) and Camacho and La Cruz (2004). Their sliding technique was derived from a first-order-plus-deadtime (FOPDT) model of the actual plant to control a class of nonlinear chemical processes. The method later has been extended for open-loop unstable processes in Rojas et al. (2004). Both these works, however, use the first order

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Taylor series to approximate the process deadtime. Sivaramakrishnan et al. (2008) later presented this technique by deriving SMC parameters using an integral squared error (ISE) criteria. The results given in Sivaramakrishnan et al. (2008) were a little improved for parameter uncertainties but not for disturbance robustness. To overcome the delay approximation problem in those methods, the Smith Predictor (SP) was adopted to eliminate deadtime together with SMC to achieve the robust controller (Camacho and La Cruz, 2004). In this work, the Smith predictor based sliding mode controller (SP-SMC) was first time evaluated for integrating processes. The robustness to parameter variation and disturbance rejection was shown to be improved compared to the original structure of SP (Camacho and La Cruz, 2004).

The aim of this paper is two-fold. First, to propose a new scheme for enhancing robustness of the system with respect to process variations and disturbance inputs. Second, to design a new control performance index for reducing control effort and improving overall regulatory performance. The limitation of the SMC design suggested in Camacho and La Cruz (2004) is overcome by providing a new discrete control law. The provided discontinuous part using relay control has physical, meaningful parameters to reduce chattering phenomena without compromising on aggressiveness to reach the sliding surface. The values of parameters in the control law have been obtained based on the minimization of Integral Squared Time Error (ISTE) criterion and measure of the total control signal variation. A new control law has been satisfied to achieve a balance trade-off between transient performance and actuator preservation. Also, it has been reported (Rajabioun, 2011; Yang and Deb, 2010) that the cuckoo optimization algorithm (COA) is superior with faster convergence and global optimal value in comparison to other search techniques. We have adopted this metaheuristic search algorithm with some modifications to satisfy the proposed control performance index.

For autotuning of the proposed SP based sliding mode control structure, process model transfer function is first obtained using a stabilized relay experiment for unstable processes. Once the process model parameters are obtained from the relay test, the parameters of the sliding mode controller can be calculated from the technique provided. Excellent performance of the proposed SP-SMC over some existing control methods is illustrated for the unstable process transfer function.

Model parameter identification

In controller design and autotuning, process identification is very important. Closed-loop tests using relay autotuning conducted online to identify the process dynamic response, have received a lot of attention from the worldwide research community and the industry (Atherton, 2006; Majhi, 2009). Normally, relay feedback is used to induce a permanent oscillation of the controlled variable. When the oscillation arises, the describing function (DF) approximation allows to identify one point of the process Nyquist curve on the basis of its frequency and amplitude. However, unstable time delay processes are mostly difficult to identify. A relay method can only be used to identify the unstable process dynamics when a limit cycle exists. We know that a limit cycle only exists when the ratio of delay (θ) to unstable time constant (τ_1) (normalized time delay $\theta_n = \theta/\tau_1$) is less than 0.693 (Majhi and Atherton, 2000b). This limitation of a relay autotuning was revisited recently by Mehta (2014) to induce a sustained oscillation at the output for large normalized time delay processes.

The following two subsections describe the procedure to estimate the unknown parameters of the unstable process transfer function model

$$G_p(s) = \frac{ke^{-\theta s}}{\tau_1 s - 1} \tag{1}$$

Here the steady state gain k, time constant τ_1 and time delay θ are unknown parameters to be estimated.

Gain identification

The steady state gain k can be obtained from the process input and output signal prior to the relay test. The steady state equation with known process input and output but unknown load disturbance is (Majhi and Atherton, 2004)

$$y(0) = -k(u(0) + d(0)) \tag{2}$$

Without load disturbance $k = -(y(0)/u(0))$. With a constant load disturbance $d(0)$, it is possible to obtain two pairs $\{u_1(0), y_1(0)\}$ and $\{u_2(0), y_2(0)\}$ from two different reference signals r_1 and r_2. Then, the steady state gain becomes

$$k = \frac{(y_1(0) - y_2(0))}{(u_1(0) - u_2(0))} \tag{3}$$

During noisy input output signal, their mean values are used in the above equation.

Time constant and time delay identification

To relax the constraint of relay feedback for unstable processes with large time delay, a technique was suggested with a PD stabilizer in Mehta (2014). Figure 1 shows the experimental setting for process identification using the relay test. An inner proportional-derivative (PD) feedback loop helps to ensure a stable limit cycle oscillation by use of a suitable choice of PD gains. A symmetrical relay with amplitudes $\pm h$ and hysteresis widths $\pm\varepsilon$ is plugged into the closed-loop system, where: r, e, u and y are the desired, control error, manipulated and controlled output variables, respectively. It is assumed that the static load disturbance d appears at the process input. When the relay autotuning is applied in the closed-loop, the process will generate a symmetrical limit cycle output as shown in Figure 2.

In general, the relay autotuning test is performed with an initial setting of PD gains $(k_c, k_d) = (1/k, 0.95/k)$. Then using the measured output peak amplitude A_p and half period T, one can estimate the unknown parameters τ_1 and θ for the unstable process model in equation (1). The detailed analytical approach to find unknown parameters of $G_p(s)$ can be found

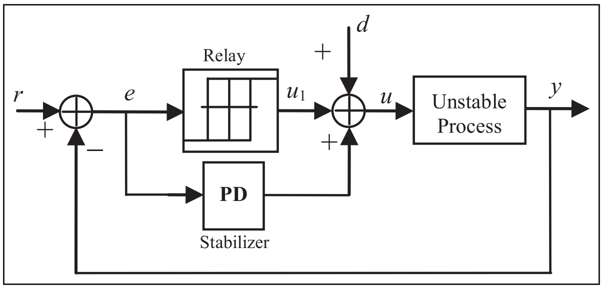

Figure 1. Structure for identification.

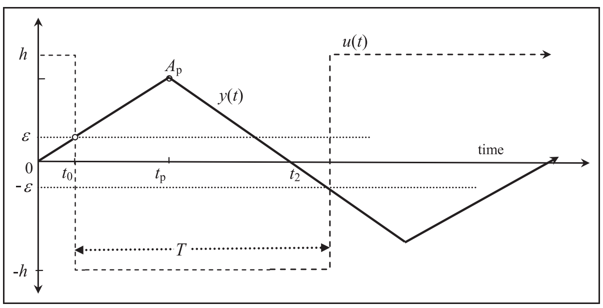

Figure 2. Limit cycle oscillation from unstable first order process.

in Mehta (2014). The explicit expressions are given in the following after performing a successful relay with hysteresis test

$$\begin{aligned} \tau_1 &= \frac{\sqrt{k^2(a_1^2 + a_2^2) - 1}}{\omega_c} \\ \theta &= \frac{tan^{-1}(a_2/a_1) + tan^{-1}(\omega_c\tau_1)}{\omega_c} \end{aligned} \tag{4}$$

where

$$\begin{aligned} a_1 &= \frac{4h}{\pi A_p^2}(\sqrt{A_p^2 - \varepsilon^2}) + kk_c \\ a_2 &= \omega_c kk_d - \frac{4h\varepsilon}{\pi A_p^2} \\ \omega_c &= \pi/T = \text{criticalfrequencywithhalfperiod}, T \end{aligned} \tag{5}$$

The detailed derivation of the above expressions is given in the Appendix. Once the process dynamics model has been estimated accurately, a new control scheme is designed in the following section using a Smith predictor combined with a sliding mode control.

Smith predictor based sliding mode control

The Smith predictor based sliding mode controller presented in this paper uses the original SP structure while the controller is a sliding mode controller (SMC). The block diagram of the proposed scheme is shown in Figure 3. Unlike the structure, for integrating processes, presented in Camacho and La Cruz (2004) that uses an additional proportional-derivative controller, a standard SP structure without additional controllers for disturbance rejection has been used.

Basic SMC design is implemented via two steps, (1) the design of a stable surface and (2) the design of a control law to force the system states onto the required surface in finite time. The designed surface is to match system uncertainties and disturbances. The initial phase when the state trajectory is directed towards a sliding surface is called the reaching phase. During the reaching phase, the system is sensitive to all types of disturbances. However, a control law can be designed which ensures finite time reaching of the sliding surface even in the presence of uncertainties and disturbances. In this work we have adopted the sliding surface, $S(t)$ to achieve the stability and tracking performance, defined by Slotine and Li (1991). The expression was defined with a characteristic of proportional, integral and differential types on the tracking error as

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e(t)dt \tag{6}$$

where e is the tracking error, n is the system order and λ is a tuning parameter to achieve the performance of the system on the sliding surface. The control law is designed such that from any initial condition, the state trajectory is forced towards the sliding surface and the trajectory remains on the surface thereafter (Bandyopadhyay et al., 2009). It should be noted that using the control law the state trajectory will hit the surface in

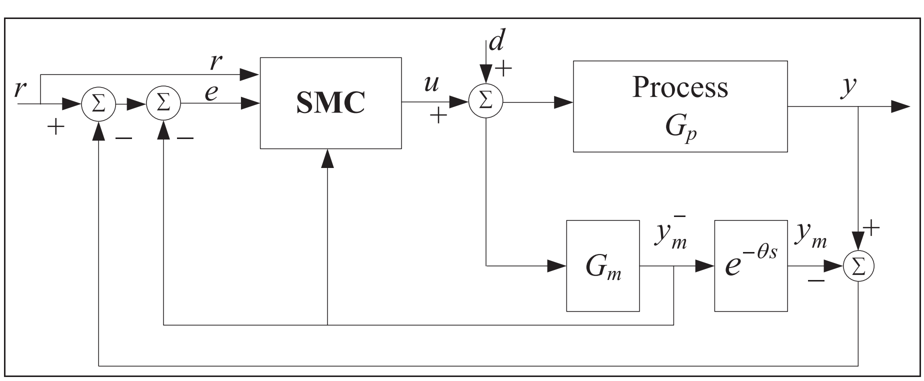

Figure 3. Proposed Smith predictor with sliding mode structure.

finite time. After hitting the sliding surface, the trajectory will slide along the surface and the system is invariant to parameter variations and external disturbances. The sliding equation (6) indicates that once $S(t)$ has reached a constant value, say $e(t) = 0$ for all $t > 0$; it is desired to set

$$\frac{dS(t)}{dt} = 0 \tag{7}$$

After defining the suitable sliding surface, the control law is required to drive the controlled variable to its setpoint value. The sliding control law, $u(t)$ consists of two additive parts: a continuous part, $u_c(t)$ and a discontinuous part, $u_d(t)$. So it gives

$$u(t) = u_c(t) + u_d(t) \tag{8}$$

The continuous part of the control law is obtained from the process input and output states (Camacho and Rojas, 2000; Utkin, 1997). In this work we have considered the continuous control part is composed of the reference value, delay free model output and error value. So we can write u_c as

$$u_c(t) = f(r(t), y_m(t), e(t)) \tag{9}$$

In contrast, the discontinuous part includes a nonlinear switching element like the ideal relay or saturation relay control. However, in practice it is difficult to implement high switching control using these ideal relay functions because of the presence of finite delay in the system or physical limitations of actuators. This causes a chattering problem around the steady state output (Camacho and Rojas, 2000; Slotine and Li, 1991; Utkin, 1997). Therefore we have considered a new type of function to reduce chattering phenomena without compromising on aggressiveness to reach the sliding surface. A control law can be obtained by the so-called power rate reaching law approach in which the switching function dynamics are specified a priori (Hung et al., 1993). This can be written as follows

$$u_d(t) = \alpha |S(t)|^{\beta} sign(S(t)) \tag{10}$$

where α and β are positive constants used to satisfy the condition in equation (7). In the sliding mode controller design, there are two main aspects, one to guide the system to the desired sliding surface and then to keep the controlled variable on the reference value. Now, for designing the continuous control part let us take a process model defined by an unstable first order transfer function as

$$G_m(s) = \frac{y_m^-(s)}{u(s)} = \frac{k_m}{\tau_m s - 1} \tag{11}$$

where k_m is the model gain and τ_m is the model time constant. Since the time delay has been isolated using a SP structure, we can ignore it in the SMC design. Then, equation (11) can be written in differential form as

$$\tau_m \frac{dy_m^-(t)}{dx} - y_m^-(t) = k_m u(t) \tag{12}$$

This gives

$$\frac{dy_m^-(t)}{dx} = \frac{1}{\tau_m}\left(k_m u(t) + y_m^-(t)\right) \tag{13}$$

Now the sliding surface is developed for the first order process model and therefore by considering $n = 1$ in equation (6) we get the proportional-integral expression as

$$S(t) = e(t) + \lambda \int_0^t e(t)dt \tag{14}$$

The above sliding surface must satisfy the condition in equation (7) and so it becomes

$$\frac{dS(t)}{dx} = \dot{e}(t) + \lambda e(t) = 0 \tag{15}$$

If we consider the regulatory problem, the constant reference value can be discarded without any variation in performance. This gives a simple expression

$$\frac{dy_m^-(t)}{dx} = \lambda e(t) \tag{16}$$

Using equations (13) and (16), the continues part of the control law is derived as

$$u_c(t) = \frac{1}{k_m}\left(\tau_m \lambda e(t) - y_m^-(t)\right) \tag{17}$$

Finally the complete form of the control signal from the SP-SMC can be generated as

$$u(t) = \frac{1}{k_m}\left(\tau_m \lambda e(t) - y_m^-(t)\right) + \alpha |S(t)|^{\beta} sign(S(t)) \tag{18}$$

with the sliding surface

$$S(t) = sign(k_m)\left((r(t) - y_m^-(t)) + \lambda \int_0^t e(t)dt\right) \tag{19}$$

The formation of above control signal, equations (18) and (19), delivers a potential benefit from the process control point of view. First, the action of the controller is guaranteed appropriately for the given process; depending on the static gain. Therefore the action never switches (Camacho and Rojas, 2000). Second, the SMC based closed loop system has a fixed structure depending on the λ parameter and process model parameters.

Optimal tuning of SP-SMC parameters

After constituting the controller equations, it is necessary to determine suitable values of tuning parameters (λ, α, β). In general, fast settling time and small overshoot are the two important aspects in most of the design problems. However, it is well-known that quick response produces a large overshoot which is not at all desirable in many practical applications. On

```
Begin

      Initialize the habitats, η= [λ,α, β] with some random points
      Define cost function
      Set COA parameters
      Initialize population and define egg lying radius for each cuckoo
   While (Iteration <= Max Iteration) or (stop criterion)
      Let cuckoos to lay eggs inside their corresponding range
      Kill eggs that are recognized by host birds
      Evaluate each cuckoo fitness/ profit values
      Limit cuckoos' maximum number in environment and kill those who live in worst
      habitats;
      If current maximum profit > global maximum profit
              Replace global maximum profit with new profit
      End

      Cluster cuckoos, find best group and select goal habitat
      Let new cuckoo population migrate toward goal habitat

   End while

      Display the iteration result with best parameters

End
```

Figure 4. Pseudo code of cuckoo optimization algorithm.

the other hand, a low overshoot can be achieved at the cost of a high settling time. Thus, the user has to choose between fast response and low overshoot and most of the design problems make a trade-off between these two transient indices.

When the process dynamics are known after the identification step, the required controller parameters may be optimized by minimizing an integral performance criterion. In particular, time moment weighted integral performance criteria gives a good step response with a small overshoot and short settling time (Zhuang and Atherton, 1993). It is known that if a control system is designed to minimize Integral Squared Time Error (ISTE) criterion then it usually gives satisfactory results. This index can be defined as follows

$$J_{ISTE}(\eta) = \int_0^\infty (te(\eta, t))^2 dt \tag{20}$$

where η denotes variable parameters which are chosen to minimize $J_{ISTE}(\eta)$. Because of the time weighting method in this formula, it penalizes the initial unavoidable errors which occur for setpoint changes (Zhuang and Atherton, 1993). But, the optimal controller by this method also may not be sufficient to claim the optimum control input signal variations. In most process control loops valves are the only components with moving parts and more variation in control signal has high costs in terms of valve wear and maintenance programs. In this context, less attention has been paid, although Skogestad (2003) has defined a different kind of criterion to measure the total variation (TV) in $u(t)$ analytically for the controller performance. If we discretize the input signal as a sequence, $[u_1, u_2, \ldots, u_i, \ldots]$, then the index

$$\mathrm{TV} = \sum_{i=1}^{\infty} |u_{i+1} - u_i| \tag{21}$$

which should be as small as possible to minimize variations in $u(t)$.

We aim to achieve superior control performance by optimizing error signal in such a way that it gives minimum controller output signal variations. We define therefore the combination of the two above indices together to balance the trade-offs between transient performance and actuator preservation as follows

$$\begin{aligned} J_{ISTE} &= \min_{\eta} \int_0^\infty (te(t))^2 dt \\ \text{subject to} : \mathrm{TV} &= \min_{\eta} \sum_{i=1}^{\infty} |u_{i+1} - u_i| \end{aligned} \tag{22}$$

In this work, we have satisfied the above performance measured using a new metaheuristic search algorithm, namely cuckoo optimization algorithm. This method has shown the superior performance with faster convergence and better global optimal achievement (Rajabioun, 2011; Yang and Deb, 2010). The performance condition in equation (22) can be optimized for a set of η values using application of this global search technique.

We redefine the performance requirement in equation (22) to build the objective function. If the controller specification is to tack the desired setpoint with optimal usage of control energy, the bi-objective design problem can be defined to build the objective function as the evaluation of profit

$$\text{Profit} = \sum_{i=1}^{n} \int_0^\infty (te(\eta, t))^2 dt + \frac{1}{\tau_m} \sum_{i=1}^{n} |u_{i+1} - u_i| \tag{23}$$

where $i \in \{1, 2, \ldots, n\}$ and τ_m is a model time constant, supposed to handle the allowable value of TV. In this work the constraint is imposed with respect to the time constant of the process, meaning a lower value of τ_m allowing some control signal variation.

The basic steps of the COA can be summarized as the pseudo code shown in Figure 4. This optimizing cuckoo search method is implemented in MATLAB 7.6 (Rajabioun,

2012) on Windows 7 core i5 Intel 4 GB RAM and computing time is less than a few minutes for the defined problem. We have tested it using a different range of parameters such as population size, maximum range of egg laying radius (ELR) and maximum number of iterations. In the original version of COA, each cuckoo has an egg laying radius which is kept within upper and lower limit range in order to represent the physical "power" of the cuckoos. It is defined in (Rajabioun (2011) as

$$\mathrm{ELR} = \Delta \times \frac{\text{No. of current cuckoo'eggs}}{\text{Total no. of eggs}} \times (\mathrm{var}_{hi} - \mathrm{var}_{low}) \quad (24)$$

where Δ is a radius coefficient, supposed to handle the maximum value of ELR. As for a radius coefficient, we experimentally found that the best performance could be obtained by initially setting Δ to some relatively high value (e.g. 3 or even higher), which corresponds to a system where cuckoos move in a low viscosity medium and perform extensive exploration, and gradually reducing Δ to a much lower value (e.g. 1 - 0.8), where the system would be more dissipative and exploitative and would be better at homing into local optima. The radius coefficient Δ is set according to the following equation

$$\Delta = \omega_{\max} - (\mathrm{it} - 1)\left(\frac{\omega_{\max} - \omega_{\min}}{\mathrm{it}_{\max} - 1}\right) \quad (25)$$

where 'it' is a current iteration number, '$\mathrm{it}_{\max}$' is a maximum iteration number and ($\omega_{\max}$, $\omega_{\min}$) are integer values. In the present paper, population size = 7, $var_{low} = 0.1$, $var_{hi} = 3$, ($\omega_{\max}$, $\omega_{\min}$) = (3,1), number of eggs laid between 2 – 8 eggs, are used to find the best SMC parameters (λ, α, β). The stopping criterion can be defined in two ways, either using a given tolerance or a fixed number of iterations. From the implementation point of view, a fixed number of iterations is easy to implement and we have set the fixed number of iterations as 70, which is sufficient for our problem of optimization. In the present approach, sliding mode control ensures the good results, as we can see from later simulations.

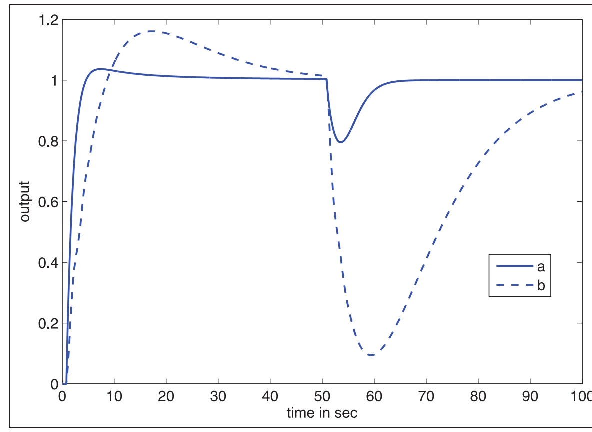

Figure 5. Step responses for Example 1: (a) proposed SP-SMC method and (b) SMC method by Sivaramakrishnan et al. (2008).

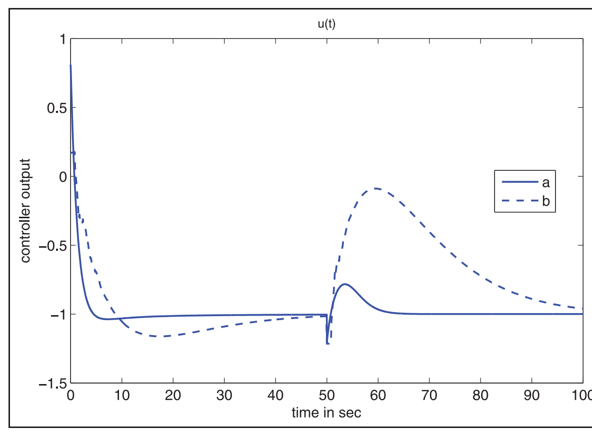

Figure 6. Controller responses for Example 1: (a) proposed SP-SMC method and (b) SMC method by Sivaramakrishnan et al. (2008).

Simulation study

Combining the process estimation method with controller design, an auto-tuner can be given. In this paper both identification and control technique are demonstrated for a class of unstable processes. The parameter variation and disturbance effects are also considered in this study. It is important to keep the output oscillation amplitude in the prescribed limit as per the tolerable process variable swing and decide values for the relay heights that produce a limit cycle with acceptable amplitude level. To overcome the undesirable relay chattering caused by noisy signals, the width of the hysteresis of the relay is set to twice the standard deviation of the noise. For ease in simulation study, the relay with $h = \pm 0.25$ and $\varepsilon = \pm 0.05$ are taken, although fairly low values of relay height could be used.

Example 1 Consider an unstable process $G_p(s) = e^{-0.8s}/(s-1)$ studied in Sivaramakrishnan et al. (2008). Initially, $k = 1$ was estimated using the method discussed in Section 2.1. By carrying out the relay feedback test from the simulation values, $A_p = 0.8109$ and $T = 1.0710$ were measured from a single test. Using (30), the estimated process model parameters were $\tau_1 = 0.9979$ and $\theta = 0.7997$, respectively. The SP-SMC parameters, (λ, α, β) are calculated as $(0.7109, 0.1, 2.4154)$. With these controller setting, the response of the closed-loop system to a unit setpoint change and a disturbance with magnitude of -0.2 at $t = 40$ s is given in Figure 5. The superior performance of the Sivaramakrishnan et al. (2008) method was reported and compared with the method in Rojas et al. (2004). For comparison, Figure 5 illustrates the closed loop response of the sliding mode control method presented by Sivaramakrishnan et al. (2008). Additionally, the proposed method resulted in the desired performance in less control signal variations with TV = 3.53, while the method of Sivaramakrishnan et al. (2008) gave, TV = 4.11. Figure 6 shows the enhanced performance of the proposed technique with good subsequent control performance.

To show the robust performance of the control design, perturbations of $\pm 20\%$ in the parameters θ, τ_1 and k of the nominal process are assumed. The robust responses are compared

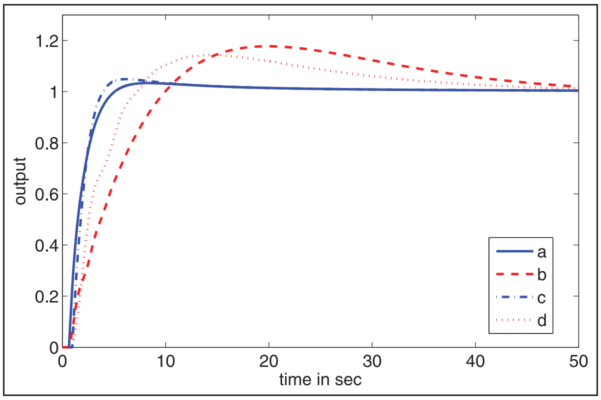

Figure 7. Robust responses when perturbation in θ: (a) proposed method for -20% change, (b) Sivaramakrishnan et al. (2008) method for -20% change, (c) proposed method for $+$ 20% change and (d) Sivaramakrishnan et al. (2008) method for $+$ 20% change.

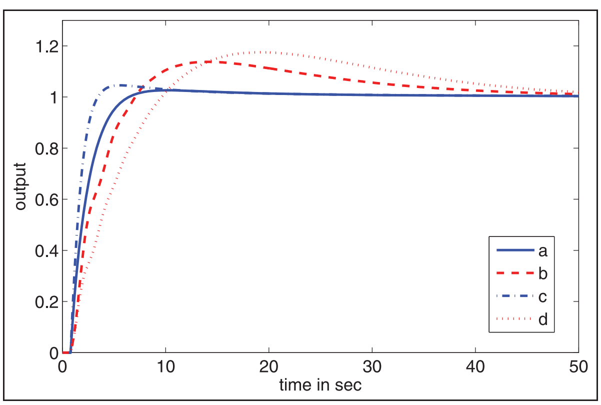

Figure 8. Robust responses when simultaneous perturbations in τ_1 and k: (a) proposed method for -20% change, (b) Sivaramakrishnan et al. (2008) method for -20% change, (c) proposed method for $+$ 20% change and (d) Sivaramakrishnan et al. (2008) method for $+$ 20% change.

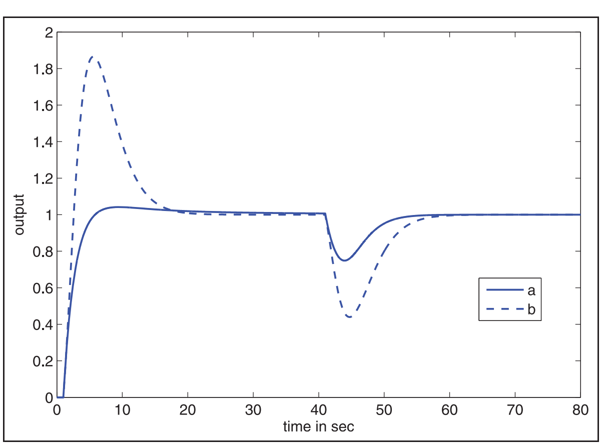

Figure 9. Step responses for Example 2: (a) proposed SP-SMC method and (b) Shamsuzzoha's PI controller.

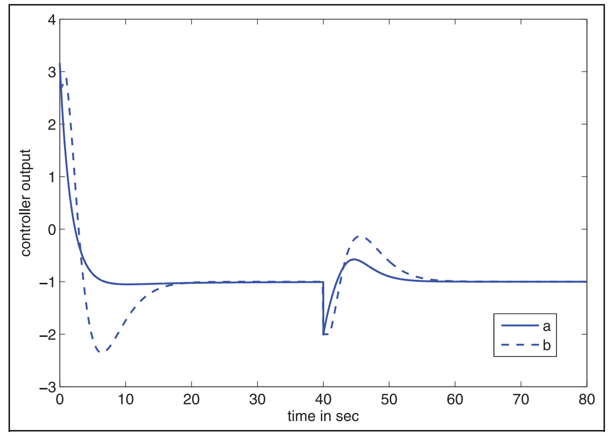

Figure 10. Controller responses for Example 2: (a) proposed SP-SMC method and (b) Shamsuzzoha's PI controller.

with the reported method in Figures 7 and 8. It is observed that the closed-loop performances are very robust by the given method.

From the TV values and the performances in Figures 7 and 8, the proposed SP-SMC scheme shows smaller control effort and robust behavior. This is beneficial for use in real equipment.

Example 2 Let the unstable process (Shamsuzzoha and Skogestad, 2010) be, $G_p(s) = e^{-s}/(5s - 1)$. From the identification procedure depicted in Section 2, the exact model $G_m(s) = e^{-1.001s}/(4.988s - 1)$ was estimated for controller design. The required SP-SMC parameters, (λ, α, β) were obtained as (0.4653, 0.8383, 2.6793) using optimizing cuckoo search method. Shamsuzzoha and Skogestad (2010) have suggested a PI controller as $(2.636 + 1/10.54s)$. The magnitude of the load disturbance is $d = -1$. The responses of the closed-loop system for these controller settings are given in Figure 9. It is obvious from the simulation results that SP-SMC controller improved system responses in terms of overshoots and settling times when a disturbance occurred in the system. Further, the controller outputs are given in Figure 10. The measure of control signal variations for the presented method was also less, 10.28 whereas their method gave TV = 13.21.

Example 3 This example considers an unstable plant with second order transfer function $G_p(s) = e^{-0.5s}/(s^2 + 1.5s - 1)$, studied by Nema and Padhy (2015).

The method recently reported by Nema and Padhy (2015) uses an ideal relay test to identify the second order process model as $1.0e^{-0.5004s}/((1.998s + 1)(0.5004s + 1))$. Based on this identified process model and using COA, they have determined the PI-PD controller parameters, $K_p = 0.78$, $T_i = 1.65$, $K_f = 1.87$, and $T_d = 0.72$. This method gave an improved result compared to previously reported methods in literature. For the same process, the analytical method for PI-PD tuning was advised firstly by Majhi and Atherton (2000b) and the controller gains were $K_p = 0.937$, $T_i = 1.399$, $K_f = 2.328$, and $T_d = 0.530$.

To prove the efficacy of the proposed design, the results were evaluated with uniformly distributed random noise of

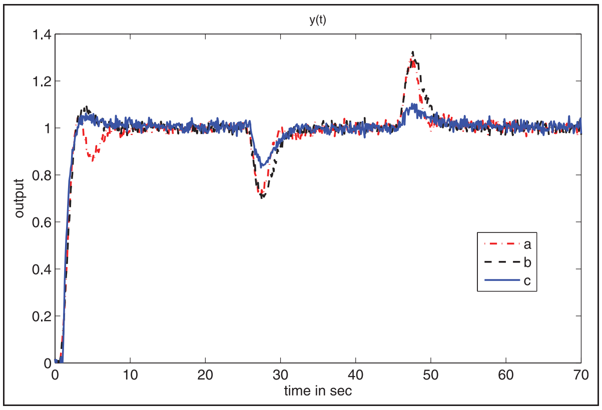

Figure 11. Responses for Example 3 with SNR 20%: (a) Majhi's PI-PD design, (b) Nema's PI-PD design and (c) proposed SP-SMC design.

signal-to-noise ratio 20 dB. Since, in practice measurement noise falling into the high frequency range of the signal spectrum, measurements are not noise-free. This noise is large enough when we consider the practical circumstances. Here, a hysteresis of the relay is more than the noise magnitude to prevent a severe fluctuation of the relay output. The identified process model parameters from the noisy output were $k = 1.001$, $\tau_1 = 2.0$ and $\theta = 1.0587$ using the depicted method. The SP-SMC parameters, (λ, α, β) were obtained as (1.1, 1.1, 2.986) and those were determined using the improved COA technique given in Section 4. The closed-loop performances for all three design methods were simulated assuming a unit step setpoint change and a noise with power of 20 dB. Results are given in Figure 11 to show the impact of the added noise on the quality of the output. Responses to disturbance with large magnitudes $d = -0.5$ at $t = 25$ s and $d = 0.5$ at $t = 45$ s, are also illustrated in the figure. It is clear that the proposed technique has a lower overshoot and results in a more smooth transition process.

Conclusions

A robust and effective control scheme has been presented for a new Smith predictor with sliding control assuming a transfer function with time delay for unstable processes. Two important advantages of the new scheme are that the disturbance rejection capabilities across the desired surface and the satisfied bi-objective performance measures with optimal input usage. The method has been shown to work effectively when the process dynamics are changed. Investigations clearly reveal that the optimum parameters obtained at the nominal condition are quite robust and need not be reset for changed conditions. Illustrative examples have been given to demonstrate the simplicity with merits of the proposed method.

Acknowledgement

The author is thankful to the University of the South Pacific for providing excellent research facilities and environment at the Electrical and Electronics Engineering department.

Conflict of interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The work by Rubén Rojas was partially supported by the Prometeo Project of the Secretariat for Higher Education, Science, Technology and Innovation of the Republic of Ecuador.

References

Ajmeri M and Ali A (2015) Two degree of freedom control scheme for unstable processes with small time delay. *ISA Transactions* 56: 308–326.

Atherton DP (2006) Relay autotuning: An overview and alternative approach. *Industrial & Engineering Chemisty Research* 45: 4075–4080.

Bandyopadhyay B, Deepak F and Kim KS (2009) *Sliding Mode Control Using Novel Sliding Surfaces*. Springer Science & Business Media, Springer Berlin Heidelberg.

Camacho O and La Cruz FD (2004) Smith predictor based-sliding mode controller for integrating processes with elevated deadtime. *ISA Transactions* 43: 257–270.

Camacho O and Rojas R (2000) A general sliding mode controller for nonlinear chemical processes. *Journal of Dynamic Systems, Measurement and Control* 122: 650–655.

Hung JY, Gao W and Hung JC (1993) Variable structure control: A survey. *IEEE Transactions on Industrial Electronics* 40(1): 2–22.

Hwang C and Hwang JH (2004) Stabilisation of first-order plus dead-time unstable processes using PID controllers. *IET Control Theory & Applications* 151(1): 89–94.

Lee SC and Wang QG (2010) Stabilization conditions for a class of unstable delay processes of higher order. *Journal of the Taiwan Institute of Chemical Engineers* 41: 440–445.

Majhi S (2009) *Advance Control Theory - A Relay Feedback Approach*. Cengage Learning Asia, Singapore.

Majhi S and Atherton D (2000a) Obtaining controller parameters for a new smith predictor using auto tuning. *Automatica* 36: 1651–1658.

Majhi S and Atherton DP (2000b) Online tuning of controllers for an unstable FOPDT process. *IEE Proceedings - Control Theory and Applications* 147(4): 421–427.

Majhi S and Atherton DP (2004) On-line tuning of controllers for unstable processes. In: *9th Portugese Conference on Automatic Control*, June 7–11, Faro, Portugal.

Matausek M and Ribic A (2012) Control of stable, integrating and unstable processes by the modified smith predictor. *Journal of Process Control* 22(1): 338–343.

Matausek MR and Micic AD (1996) A modified smith predictor for controlling a process with an integrator and long dead-time. *IEEE Transactions on Automatic Control* 41(8): 1199–1203.

Mehta U (2014) Stabilizing relay experiment for modeling of unstable processes with large time delay. *IMechE Part I- Journal of Systems and Control Engineering* 228(6): 359–368.

Nema S and Padhy PK (2015) Identification and cuckoo PI-PD controller design for stable and unstable processes. *Transactions of the Institute of Measurement and Control* 37(6): 708–720

Padhan DG and Majhi S (2012) Modified smith predictor based cascade control of unstable time delay processes. *ISA Transactions* 51(1): 95–104.

Perruquetti W and Barbot JP (2002) *Sliding Mode Control in Engineering, Automation and Control Series*. New York: Marcel Dekker Inc.
Rajabioun R (2011) Cuckoo optimization algorithm. *Applied Soft Computing* 11: 5508–5518.
Rajabioun R (2012) Cuckoo Optimization Algorithm, *MATLAB Central File Exchange, http://www.mathworks.com/matlabcentral/fileexchange/35635-cuckoo-optimization-algorithm*. Retrieved August 2, 2014.
Rojas R, Camacho O and Gonzalez L (2004) A sliding mode control proposal for open-loop unstable processes. *ISA Transactions* 43: 243–255.
Shamsuzzoha M and Skogestad S (2010) The setpoint overshoot method: A simple and fast closed-loop approach for PID tuning. *Journal of Process Control* 20(10): 1220–1234.
Sivaramakrishnan S, Tangirala AK and Chidambaram M (2008) Sliding mode controller for unstable systems. *Chemical and Biochemical Engineering Quarterly* 22(1): 41–47.
Skogestad S (2003) Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control* 13: 291–309.
Slotine JJ and Li W (1991) *Applied Nonlinear Control*. Prentice-Hall, Englewood Cliffs, New Jersey.
Sree RP and Chidambaram M (2005) Set point weighted PID controllers for unstable systems. *Chemical Engineering Communications* 192(1): 1–13.
Utkin VI (1997) Variable structure system with sliding mode. *IEEE Transactions on Automatic Control* 22(2): 212–221.
Yang XS and Deb S (2010) Engineering optimization by cuckoo search. *International Journal of Mathematical Modelling and Numerical Optimisation* 4: 330–343.
Zhuang M and Atherton DP (1993) Automatic tuning of optimum PID controllers. *IEE Proceedings - Control Theory and Applications* 140: 216–224.

Appendix

The detailed derivations of equations (4) and (5) are given in this Appendix. According to well-known describing function theory, a relay test will shift the limit cycling point on the Nyquist curve to a new location where the phase lag is less than -π. The critical frequency ω_c of a process and a self-sustained oscillation of peak amplitude, A_p can be determined from the induced limit cycle oscillations. In this experiment, a PD controller $C(s) = (k_c + k_d s)$ is connected in parallel to the relay and it induces the limit cycle oscillations at the output with frequency ω_c when

$$G(j\omega_c)[N(A_p) + C(j\omega_c)] = -1 \tag{26}$$

where

$$N(A_p) = \frac{4h}{\pi A_p^2}\left(\sqrt{A_p^2 - \varepsilon^2} - j\varepsilon\right) \tag{27}$$

is the DF of the relay with hysteresis. Substitution of $G(j\omega_c) = \frac{ke^{-j\omega_c\theta}}{j\omega_c\tau_1 - 1}$ and $C(j\omega_c) = (k_c + j\omega_c k_d)$ in (26) gives

$$\frac{ke^{-j\omega_c\theta}}{(j\omega_c\tau_1 - 1)}(a_1 + ja_2) = -1 \tag{28}$$

where

$$\begin{aligned} a_1 &= \frac{4h}{\pi A_p^2}\left(\sqrt{A_p^2 - \varepsilon^2}\right) + kk_c \\ a_2 &= \omega_c k k_d - \frac{4h\varepsilon}{\pi A_p^2} \\ \omega_c &= \pi/T = \text{critical frequency with half period, } T \end{aligned} \tag{29}$$

By equating the real and imaginary parts of both sides of equation (28), we obtain explicit expressions for τ_1 and θ as

$$\begin{aligned} \tau_1 &= \frac{1}{\omega_c}\sqrt{k^2(a_1^2 + a_2^2) - 1} \\ \theta &= \frac{\tan^{-1}(a_2/a_1) + tan^{-1}(\omega_c\tau_1)}{\omega_c} \end{aligned} \tag{30}$$